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Decoupling control law for structural control implementation

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Abstract

Multi-story buildings, subjected to wind or earthquake excitation, can be modeled as multi-degree of freedom (MDOF) systems defined by a set of coupled second order ordinary differential equations. In this paper, the dynamic coupling characteristics of multi-story building are examined, and it is found that the coupled property in a system can be described as a positive feedback from the control theory point of view. This positive feedback property of a MDOF system may intensify structural vibration. For the structural control implementation, open-loop and closed-loop decoupling control laws are proposed. All coupled "channels" of the system are "broken off" when the vibration control design is based on the proposed control laws. A complex MDOF structural system, therefore, is equivalent to a set of single degree of freedom (SDOF) systems, and the control design can be carried out independently for any specific degree of freedom. Thus, the proposed control laws provide an efficient tool by which the vibration of a selected floor can be suppressed without any effect on its neighboring floors because the control is one to one. Meanwhile, the computational procedure of the control design can be significantly simplified because all analyses and design are conducted based on SDOF systems.

Keywords: Structural control; Vibration; Dynamics; Earthquake excitation

1. Introduction

In civil engineering, the research in active control of structural vibration has been conducted for more than 20 years, starting in the 1970s (e.g. Yao, 1972). Although work on this area is relative recent, significant progress, both in theoretical and experimental, has been achieved (e.g., Chang and Soong, 1980; Hrovat et al., 1983; Samali et al., 1985; Soong, 1988; Burdisso, 1994; Mukai et al., 1994; Li and Reinhorn, 1995; Köse et al., 1996; Wu and Soong, 1996; Li et al., 1999, 2000). A variety of active control laws has been developed specifically for civil engineering, and a number of full-scale buildings are currently implemented with active control systems. For example, more than 20 buildings have been installed active control devices in Japan, primarily to enhance occupant comfort during periods of high winds. As pointed out by

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Housner et al. (1997), structural control is an important part of designing new structures and retrofitting existing structures to mitigate the effects of earthquakes and strong winds. Generally speaking, the objectives of structural control are of:

- Reduce inter-story drifts for protecting the safety and integrity of the building subjected to strong earthquakes.
- Suppress floor accelerations for enhancing the occupant comfort when the building is subjected to strong wind gusts.
- Reduce the acceleration of the floors on which the equipment is installed for protecting the secondary structures in the buildings, such as nonstructural components, vibration-sensitive equipment etc.

Most work to date has concentrated on reducing the displacements and accelerations of the floors of buildings. For satisfying the first two requirements stated above, some methods of structural control have been used successfully. For the last case, it is required that control laws not only reduce the acceleration and displacement responses of a selected floor in a specified range, but also do not affect the responses of the rest floors. However, in general, most current control design is based on the entire structure, and may not satisfy this control requirement due to the coupled property of the system. For a coupled system, the "output" of a given "channel" is related to all control "channels". To control each "channel" independently, the system must be decoupled so that the control is one to one. The statement that "control is one to one" means that the output of one channel is only related to the "input" of this channel. Recently, Fang et al. (1997, 2000) studied the decoupling control law based on the independent modal space control (IMSC) algorithm. They proposed a modified independent modal space control (MIMSC) algorithm, which overcomes several shortcomings of the IMSC, and significantly simplifies the computational procedure for vibration control design. However, the proposed decoupling procedure is performed in modal space, results in the coupling still existing in physical space, i.e., the decoupling is incomplete. In order to realize complete decoupling of the system in physical space, an implementable decoupling control law, by which the one to one control can be realized in physical space, is developed in this paper. Firstly, the general coupled property of a MDOF structural system and its effects on the response are examined and discussed. It is pointed out that the dynamic coupling of a system is equivalent to positive feedback from the control theory point of view. Secondly, two types of decoupling laws, the open-loop and the closed-loop decoupling control law, are proposed. The proposed control laws treat a MDOF system as a set of equivalent independent SDOF systems. The control design and structural response analysis of the structure control system can be conducted following the same procedure as for the equivalent SDOF systems. Therefore, one to one control is realized. Finally, the control design is performed for a six-story building subjected to base excitation. The numerical simulation results indicate that the displacement and acceleration of the structure are reduced significantly, and the responses of the selected floor can be controlled independently using the proposed closed-loop decoupling control law.

It should be noted that the focus of this study does not include consideration of actuator dynamics, which can be important, depending on the specific actuator employed and the bandwidth of the actuator dynamics. Further research is being carried out towards decoupling control problems including actuator dynamics.

2. Canonical description of coupling

A multi-story structure is modeled as a MDOF system in this paper. The equation of motion is

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F}(t) \tag{1}$$

where $\mathbf{X} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ and $\mathbf{F}(t) = [f_1(t), f_2(t), \ldots, f_n(t)]^{\mathrm{T}}$ are n-dimensional displacement and external loading vectors, respectively. $\mathbf{M} = n \times n$ diagonal matrix with the ith diagonal element m_i; $\mathbf{C}$ and $\mathbf{K} = n \times n$ tridiagonal damping and stiffness matrices, respectively. Superscript T denotes vector or matrix transpose. Let

$$\mathbf{D} = \mathrm{diag}(c_i + c_{i+1}) \qquad \mathbf{E} = \mathrm{diag}(k_i + k_{i+1}) \tag{2}$$

where $i = 1, 2, \ldots, n$. $c_{n+1} = 0$ and $k_{n+1} = 0$. (The same definitions are given in the rest parts of this paper.)

Eq. (1) can now be rewritten as

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{D}\dot{\mathbf{X}} + \mathbf{E}\mathbf{X} = \mathbf{F}(t) + \mathbf{L}\dot{\mathbf{X}} + \mathbf{H}\mathbf{X} \tag{3}$$

where

$$\mathbf{L} = \begin{bmatrix} 0 & c_2 & \cdots & 0 \\ c_2 & 0 & \cdots & \cdots \\ \cdots & \cdots & 0 & c_n \\ \cdots & \cdots & c_n & 0 \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} 0 & k_2 & \cdots & 0 \\ k_2 & 0 & \cdots & \cdots \\ \cdots & \cdots & 0 & k_n \\ \cdots & \cdots & k_n & 0 \end{bmatrix}$$

Applying Laplace transform to the both sides of Eq. (3), yields

$$\mathbf{M}s^2\mathbf{X}(s) + \mathbf{D}s\mathbf{X}(s) + \mathbf{E}\mathbf{X}(s) = \mathbf{F}(s) + \mathbf{L}s\mathbf{X}(s) + \mathbf{H}\mathbf{X}(s) \tag{4a}$$

or

$$\overline{\mathbf{V}}(s)\mathbf{X}(s) = \mathbf{F}(s) + \mathbf{P}(s)\mathbf{X}(s) \tag{4b}$$

where

$$\overline{\mathbf{V}}(s) = \mathbf{M}s^2 + \mathbf{D}s + \mathbf{E} \qquad \mathbf{P}(s) = \mathbf{L}s + \mathbf{H} \tag{5}$$

Rewriting Eq. (5) in an explicit form

$$\overline{\mathbf{V}}(s) = \mathrm{diag}(m_i s^2 + (c_i + c_{i+1})s + (k_i + k_{i+1})) \tag{6a}$$

$$\mathbf{P}(s) = \begin{bmatrix} 0 & c_2 s + k_2 & \cdots & 0 \\ c_2 s + k_2 & 0 & \cdots & \cdots \\ \cdots & \cdots & \cdots & c_n s + k_n \\ 0 & \cdots & c_n s + k_n & 0 \end{bmatrix} \tag{6b}$$

Let $\mathbf{V}(s) = [\overline{\mathbf{V}}(s)]^{-1}$. By Eq. (4b), yields

$$\mathbf{X}(s) = \mathbf{V}(s)\mathbf{F}(s) + \mathbf{V}(s)\mathbf{P}(s)\mathbf{X}(s) \tag{7a}$$

or

$$x_i(s) = v_{ii}(s)f_i(s) + v_{ii}(s)\sum_{\substack{k=1 \\ k\neq i}}^{n} p_{ik}(s)x_k(s) \quad (i = 1, 2, \ldots, n) \tag{7b}$$

where $v_{ii}(s)$ and $p_{ik}(s)$ are the elements of the matrices $\mathbf{V}(s)$ and $\mathbf{P}(s)$, respectively.

The transfer function of the system is

$$\mathbf{T}(s) = [\mathbf{I} - \mathbf{V}(s)\mathbf{P}(s)]^{-1}\mathbf{V}(s) \tag{8}$$

Eq. (7b) illustrates that the response of the ith floor of the structure depends not only on the external loading acting on it but also on the responses of the other floors. In other words, the "output" x_i of the ith "channel" of a structure is not only influenced by the "input" of the ith "channel", but also by the "outputs" of its neighboring "channels". A system with such coupling property was defined as V-canonical

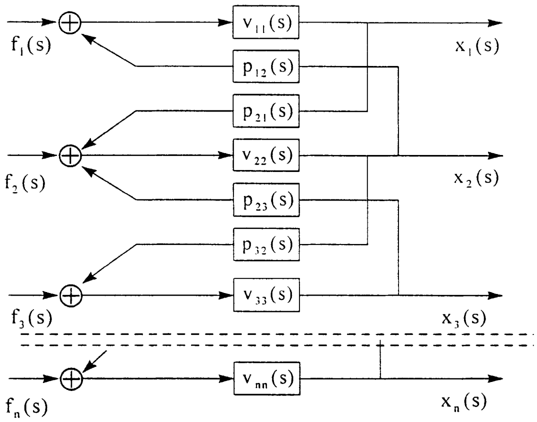


Fig. 1. The V-canonical description of multi-story structure.

plant by Mesarovic (1960), and can be illustrated in Fig. 1. It is noticed from Fig. 1 that the second term of the right-hand side of Eq. (7b) is positive feedback of x_k $(k \neq i)$. Therefore, it can be concluded that the coupling of the structural system may amplify the structural vibration. As pointed out by Liu (1983), the canonical form of the coupling has not only a mathematical meaning, but also a practical meaning. This relationship affects the decoupling design in practice. When the coupling form has been identified, then the form of decoupling elements can be determined so as to obtain the simplest decoupling conditions and make the decoupling elements be most easily realized.

3. Open-loop decoupling control law

A system with n inputs and n outputs is said to be decoupled if and only if its transfer function is given by

$$\mathbf{T}(s) = \begin{bmatrix} h_{11}(s) & & & 0 \\ & h_{22}(s) & & \\ & & \cdots & \\ 0 & & & h_{nn}(s) \end{bmatrix} = \mathrm{diag}(h_{ii}(s)) \tag{9}$$

where $h_{ii}(s)$ is not zero, i.e., for a decoupled system, the output of the ith channel is only influenced by the ith input, and is not related to any other "channels". Therefore, the objective of decoupling in control design is to find a control strategy which makes the transfer matrix of the controlled system become a diagonal matrix.

For the transfer function matrix $\mathbf{T}(s)$ which is given by Eq. (8), $\mathbf{I}$ is a unit matrix and $\mathbf{V}$ is a diagonal matrix. The transfer function matrix $\mathbf{T}(s)$ will become a diagonal matrix if the following condition is satisfied.

$$p_{ij}(s) = 0 \quad (i \neq j) \tag{10}$$

Eq. (6b) indicates that $\mathbf{P}(s) = \mathbf{0}$ if $p_{ij} = 0$ $(i \neq j)$, but the entries p_{ij} of $\mathbf{P}(s)$ represent the coupling relationship of each DOF. Therefore, $\mathbf{P}(s) = \mathbf{0}$ means that all coupling channels are "broken off". This conclusion is logical (Fig. 1) because the uncontrolled system does not have regulation function. If decoupling is realized, all coupling "channels" should be "broken off". Usually, the coupling "channels" can be "broken off" in two ways:

1. breaking off all connections among the floors;
2. adding compensation “channels”.

The first method evidently, can not be used since the connections are objective substances, which can not be broken off freely. Thus, only the second method can be applied to carry out the decoupling design.

Suppose that compensation “channel” is introduced. Then, the governing equation (4b) is:

$$(\mathbf{M}s + \mathbf{D}s + \mathbf{E}s)\mathbf{X}(s) = \mathbf{F}(s) + \mathbf{P}(s)\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s) \tag{11}$$

where $\mathbf{U}(s) = [u_1(s), u_2(s), \ldots, u_m(s)]^{\mathrm{T}}$ is the introduced compensation channel (control force vector), and $\mathbf{B}$ is an $n \times m$ matrix which denotes the location of the control force.

In view Eq. (11), the coupling effects can be eliminated if the following condition is satisfied

$$\mathbf{P}(s)\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s) = \mathbf{0} \tag{12a}$$

or

$$\overline{u}_i(s) = -\sum_{\substack{k=1 \\ k\neq i}}^{n} p_{ik}(s)x_k(s) \quad (i = 1, 2, \ldots, n) \tag{12b}$$

where $\overline{u}_i(s)$ are the elements of the matrix $\mathbf{BU}(s)$.

Eqs. (12a) and (12b) show that the compensation channel is a negative feedback of x_k $(k \neq i)$, in which the feedback gains are the elements of $\mathbf{P}(s)$. This control strategy can be described in Fig. 2 (for simplicity, two DOF is taken as an example).

Compare Eqs. (11) with (12a) and (12b), the control force vector $\mathbf{U}(s)$ can be expressed as

$$\mathbf{U}(s) = -\mathbf{B}^{+}\mathbf{P}(s)\mathbf{X}(s) \tag{13}$$

where $\mathbf{B}^{+}$ is pseudo-inverse matrix of $\mathbf{B}$:

$$\mathbf{B}^{+} = (\mathbf{B}^{\mathrm{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}} \tag{14}$$

The transfer function matrix of the controlled structure is

$$\mathbf{T}_{\mathrm{D}}(s) = (\mathbf{M}s^2 + \mathbf{D}s + \mathbf{E})^{-1} \tag{15}$$

Because $\mathbf{T}_{\mathrm{D}}(s)$ is a diagonal matrix, the structural system is decoupled.

Eq. (15) shows that a multi-story structure can be equivalent to a set of single story structures by means of the decoupling control design. That is, the response (output) of the ith floor of the structure is related only to the external loading (input) acting on it. Therefore, the same procedure, which is used to analyze a SDOF system, can be applied to analyze a multi-story structure for the design of structural control.

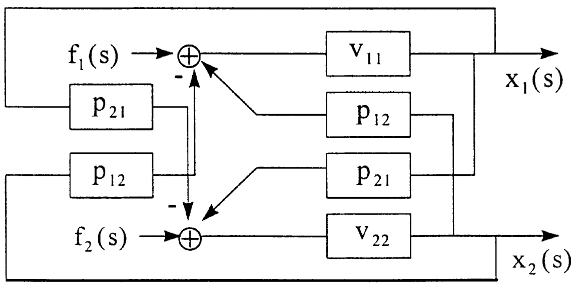


Fig. 2. The open-loop decoupling control law.

Additionally, since "breaking off" of coupled "channels" is equivalent to eliminating the positive feedback of the system, the vibration can be thus suppressed.

4. Closed-loop decoupling control

Though all coupled "channels" can be "broken off" by the open-loop decoupling control design, the decoupled elements are not equipped with any regulation function. The "output" of a selected floor, therefore, cannot be regulated by the open-loop decoupling control design. The design of a decoupling control system with a regulation function can be obtained by combining the decoupling elements with the principal regulators as described in Fig. 3.

According to Fig. 3, the response of a structure can be expressed as

$$\mathbf{X}(s) = [\mathbf{I} + \mathbf{V}(s)(\mathbf{R}(s) - \mathbf{P}(s))]^{-1}\mathbf{V}(s)\mathbf{F}(s) \tag{16}$$

where $\mathbf{R}(s)$ is a feedback matrix, and the control force vector $\overline{\mathbf{U}}(s)$ is

$$\overline{\mathbf{U}}(s) = \mathbf{B}\mathbf{U}(s) = -\mathbf{R}(s)\mathbf{X}(s) \tag{17}$$

The elements of $\mathbf{R}(s)$, $r_{ii}(s)$ and $r_{ij}(s)$ $(i \neq j)$ $(i, j = 1, 2, \ldots, n)$, are the principal regulators and the decoupling elements, respectively.

According to Eq. (9), the system is decoupled when the transfer matrix is a diagonal matrix, i.e.

$$\mathbf{T}_{\mathrm{D}}(s) = [\mathbf{I} + \mathbf{V}(s)(\mathbf{R}(s) - \mathbf{P}(s))]^{-1}\mathbf{V}(s) = \text{diagonal matrix} \tag{18}$$

In Eq. (18), since the matrix $\mathbf{V}(s)$ is a diagonal matrix, the decoupling condition can be simply expressed as

$$\mathbf{R}(s) - \mathbf{P}(s) = \text{diagonal matrix} \tag{19}$$

i.e.

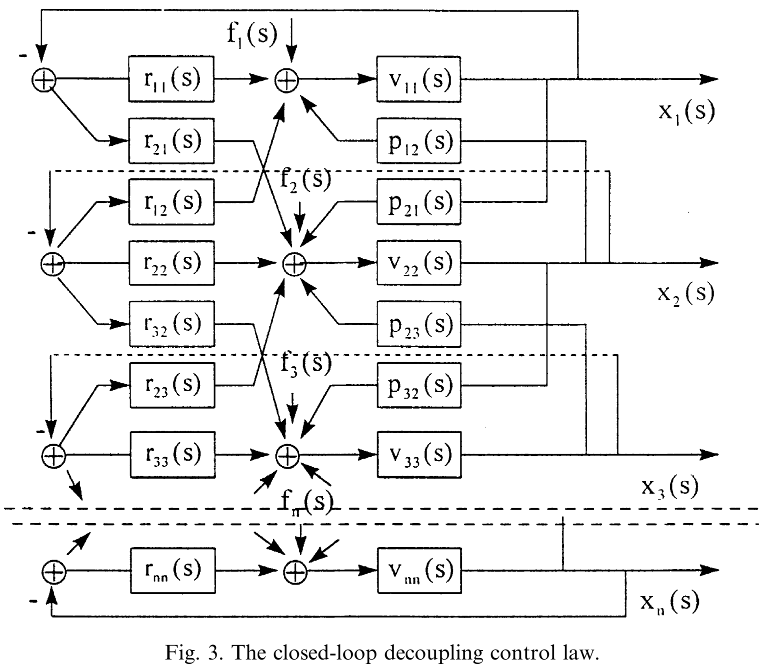


Fig. 3. The closed-loop decoupling control law.

$$r_{ii}(s) = a_{ii}(s) \qquad r_{ij}(s) = p_{ij}(s) \quad (i \neq j) \tag{20}$$

where $a_{ii}(s)$ $(i = 1, 2, \ldots, n)$ are the principal regulators which depend on the structural parameters and control algorithms.

Compare Eq. (20) with Eqs. (12a) and (12b) the decoupling elements of the closed-loop control law are the same as those of the open-loop control law. This is due to the fact that decoupling means to "break off" all connections among the "channels" regardless of whether the decoupling elements are in the open-loop or in the closed-loop control law.

The transfer matrix of the decoupled system is

$$\mathbf{T}_\mathrm{D}(s) = [\mathbf{I} + \mathbf{V}(s)\mathbf{A}(s)]^{-1}\mathbf{V}(s) \tag{21}$$

where $\mathbf{A}(s)$ is a diagonal matrix with elements $a_{ii}(s)$ $(i = 1, 2, \ldots, n)$. The motion equation of any floor is

$$x_i(s) = T_{ii}(s) f_i(s) \quad (i = 1, 2, \ldots, n) \tag{22}$$

where

$$T_{ii}(s) = \frac{v_{ii}(s)}{1 + v_{ii}(s)a_{ii}(s)} \quad (i = 1, 2, \ldots, n) \tag{23}$$

are the elements of the transfer matrix, and

$$\mathbf{R}(s) = \begin{bmatrix} a_{11}(s) & p_{12}(s) & \cdots & 0 \\ p_{21}(s) & a_{22}(s) & \cdots & \cdots \\ \cdots & \cdots & \cdots & p_{(n-1)n}(s) \\ 0 & \cdots & p_{n(n-1)} & a_{nn}(s) \end{bmatrix} \tag{24}$$

Similar to Eq. (13), the control forces can be obtained as

$$\mathbf{U}(s) = -\mathbf{B}^{+}\mathbf{R}(s)\mathbf{X}(s) \tag{25}$$

where $\mathbf{B}^{+}$ is a pseudo-inverse matrix of $\mathbf{B}$ as given in Eq. (14).

According to Eqs. (21) and (22), the motion equation of the decoupled structure is a set of independent second order ordinary differential equations, i.e. all coupling has been eliminated. Eq. (22) also shows that the design of the ith principal regulator is related only to the control algorithm and the coefficients of the ith equation, so that each regulator can be designed independently. Therefore, different control laws can be employed to design the different principal regulators in order to satisfy particular control requirements.

5. The design of the principal regulator

All principal regulators are designed as a set of PD regulators. i.e.

$$a_{ii}(s) = a_i s + b_i$$

where a_i and b_i are constants which depend on the control algorithm. The elements of the feedback matrix, then, are

$$r_{ii}(s) = a_i s + b_i \qquad r_{ij}(s) = p_{ij}(s) \quad (i \neq j) \tag{26}$$

The motion equations of the controlled structure are

$$[m_i s^2 + (c_i + c_{i+1} + a_i)s + (k_i + k_{i+1} + b_i)]x_i(s) = f_i(s) \quad (i = 1, 2, \ldots, n) \tag{27}$$

According to Eq. (27), the parameters a_i and b_i can be determined in frequency domain. Let $\mathbf{Y} = (\mathbf{X}, \dot{\mathbf{X}})^\mathrm{T}$ be the state variable, the PD regulator in frequency domain is equivalent to the feedback of the state variables in state space. The design of the principal regulator, therefore, can be also carried out in state space.

In state space, Eq. (25) can be rewritten as

$$\mathbf{U}(t) = -\mathbf{B}^{+}\mathbf{R}\mathbf{Y}(t) = \mathbf{B}^{+}\overline{\mathbf{U}}(t), \quad \mathbf{R} = [\mathbf{R}_1, \mathbf{R}_2]_{n\times 2n} \tag{28a}$$

where

$$\mathbf{R}_1 = \begin{bmatrix} b_1 & k_2 & 0 & \cdots & 0 \\ k_2 & b_2 & k_3 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & k_{n-1} & k_n \\ 0 & \cdots & 0 & k_n & b_n \end{bmatrix} \quad \mathbf{R}_2 = \begin{bmatrix} a_1 & c_2 & 0 & \cdots & 0 \\ c_2 & a_2 & c_3 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & a_{n-1} & c_n \\ 0 & \cdots & 0 & c_n & a_n \end{bmatrix} \tag{28b}$$

and $\overline{\mathbf{U}}(t) = [\overline{u}_1(t), \overline{u}_2(t), \ldots, \overline{u}_2(t)]^{\mathrm{T}}$ which can be expressed as

$$\begin{cases} \overline{u}_1(t) = -b_1x_1(t) - k_2x_2(t) - a_1\dot{x}_1(t) - c_2\dot{x}_2(t) \\ \overline{u}_2(t) = -k_2x_1(t) - b_2x_2(t) - k_3x_3(t) - c_2\dot{x}_1(t) - a_2\dot{x}_2(t) - c_3\dot{x}_3(t) \\ \cdots \quad \cdots \\ \overline{u}_i(t) = -k_ix_{i-1}(t) - b_ix_i(t) - k_{i+1}x_{i+1}(t) - c_i\dot{x}_{i-1}(t) - a_i\dot{x}_i(t) - c_{i+1}\dot{x}_{i+1}(t) \\ \cdots \quad \cdots \\ \overline{u}_n(t) = -k_nx_{n-1}(t) - b_nx_n(t) - a_n\dot{x}_n(t) - c_n\dot{x}_{n-1}(t) \end{cases} \tag{29}$$

It follows Eq. (28a) that the control force vector $\mathbf{U}(t)$ can be obtained. It is clear from Eq. (29) that the feedback gain matrix of control force is the function of stiffness and damping. It seems that the control force will be extremely large. However, consider the phase difference between the displacement and velocity as well as between the neighbor floors, the control law will not produce too large control force. The numerical examples will clarify this issue.

The decoupled motion equation of the ith floor is

$$m_i\ddot{x}_i + (c_i + c_{i+1})\dot{x} + (k_i + k_{i+1})x = f_i + V_i \tag{30a}$$

$$V_i = -a_i\dot{x}_i - b_ix_i \tag{30b}$$

Rewriting Eq. (30a) in the form of a state space equation

$$\dot{\mathbf{Y}}_i = \mathbf{A}_i\mathbf{Y}_i + \mathbf{B}_1\mathrm{V}_i + \mathbf{D}_1f_i \tag{31a}$$

where $\mathbf{Y}_i = (x_i, \dot{x}_i)^{\mathrm{T}}$ and

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 \\ -(k_i + k_{i+1})/m_i & -(c_i + c_{i+1})/m_i \end{bmatrix} \quad \mathbf{B}_1 = \mathbf{D}_1 = \begin{bmatrix} 0 & \dfrac{1}{m_i} \end{bmatrix}^{\mathrm{T}} \tag{31b}$$

The classical linear quadratic regulator (LQR) control law can be used to determine V_i, and the quadratic performance index J is

$$J = \int_0^{t_f} \left(\mathbf{Y}_i^{\mathrm{T}}\mathbf{Q}_i\mathbf{Y}_i + R_iV_i^2\right)\mathrm{d}t \tag{32}$$

where $\mathbf{Q}_i$ is a positive definite or semi-positive weight matrix, and $R_i > 0$. By minimizing J, V_i can be obtained as

$$V_i(t) = -\frac{1}{R_i}\mathbf{B}_1^{\mathrm{T}}\mathbf{P}_i\mathbf{Y}_i(t) \tag{33}$$

When the control time t_f is long enough, the matrix $\mathbf{P}_i$ satisfies algebra Riccati equation as follows

$$\mathbf{P}_i\mathbf{A}_i + \mathbf{A}_i{}^{\mathrm{T}}\mathbf{P}_i - \frac{1}{R_i}\mathbf{P}_i\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}}\mathbf{P}_i + \mathbf{Q}_i = \mathbf{0} \tag{34}$$

When the weight matrix $\mathbf{Q}_i$ is chosen as a diagonal matrix diag (Q_{1i}, Q_{2i}), and $\mathbf{P}_i$ is rewritten as

$$\mathbf{P}_i = \begin{bmatrix} p_{1i} & p_{2i} \\ p_{2i} & p_{3i} \end{bmatrix},$$

$V_i(t)$ is given by

$$V_i(t) = -\frac{p_{2i}}{R_i m_i}x_i(t) - \frac{p_{3i}}{R_i m_i}\dot{x}_i(t) \tag{35a}$$

and the solutions of Eq. (34) are

$$p_{2i} = R_i m_i(k_i + k_{i+1})\left[\sqrt{1 + \frac{Q_{1i}}{R_i(k_i + k_{i+1})^2}} - 1\right] \tag{35b}$$

$$p_{3i} = R_i m_i(c_i + c_{i+1})\left[\sqrt{1 + \frac{2p_{2i} + Q_{2i}}{R_i(c_i + c_{i+1})^2}} - 1\right] \tag{35c}$$

Compare Eq. (30b) with Eqs. (35a)–(35c), a_i and b_i are given by

$$a_i = (c_i + c_{i+1})[\gamma_i - 1] \tag{36a}$$

$$b_i = (k_i + k_{i+1})[\beta_i - 1] \tag{36b}$$

where

$$\gamma_i = \sqrt{1 + \frac{2p_{2i} + Q_{2i}}{R_i(c_i + c_{i+1})^2}} > 1 \tag{36c}$$

$$\beta_i = \sqrt{1 + \frac{Q_{1i}}{R_i(k_i + k_{i+1})^2}} > 1 \tag{36d}$$

The motion equations of the controlled structure are

$$m_i\ddot{x}_i + \beta_i(c_i + c_{i+1})\dot{x}_i + \gamma_i(k_i + k_{i+1})x_i = f_i \quad (i = 1, 2, \ldots, n) \tag{37}$$

Using Eqs. (36a)–(36d), (30b), (28a) and (28b), the control forces can be obtained.

6. Numerical example

A six-story building is modeled as a MDOF system as shown in Fig. 4. The lumped mass at each floor is $m_i = m = 3.456 \times 10^5$ kg, the stiffness of each story is $k_i = k = 3.405 \times 10^5$ KN/m and the internal damping coefficient of each story is $c_i = c = 2937$ ton/s $(i = 1, 2, \ldots, 6)$. The number of the control forces is 6. For two types of base excitations to be described below, the numerical simulation for the control design is carried out as follows.

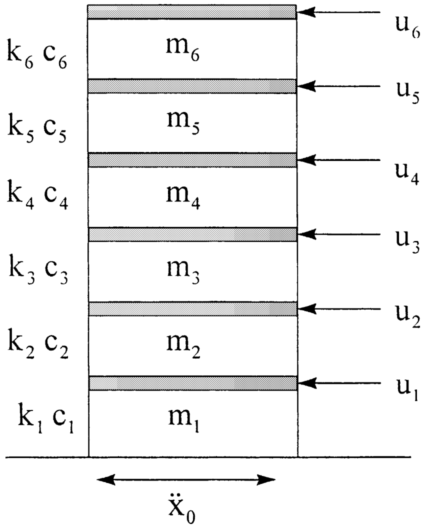


Fig. 4. The model of a six-story building.

6.1. Example 1: band-limited white noise

The base excitation is modeled as a band-limited Gaussian white noise with intensity $S_0 = 0.0159$ m^2/s^3 and bandwidth is 10 Hz. A simulated 300 s time histories of the base excitation is shown in Fig. 5.

The closed-loop decoupling control law is used to carry out the control design for this structure. The weight matrices are

$$\mathbf{Q}_i = \begin{bmatrix} 1.3 \times 10^8 & \\ & 1.3 \times 10^7 \end{bmatrix}, \qquad R_i = 10^{-5} \quad (i = 1, 2, \ldots, 6) \tag{38}$$

By Eqs. (35a)–(35c) and (36a)–(36d) the principal regulator parameters γ_i and β_i are obtained as

$$\begin{aligned} &\gamma_i = 1.0, \qquad \beta_i = 1.0187 \quad (i = 1, 2, \ldots, 5) \\ &\gamma_6 = 1.0001, \qquad \beta_6 = 1.0734 \end{aligned} \tag{39}$$

For comparison purpose, LQR is used to re-design the controllers. The performance index is

$$J = \int_0^\infty (\mathbf{X}^{\mathrm{T}}\mathbf{Q}\mathbf{X} + \mathbf{U}^{\mathrm{T}}\mathbf{R}\mathbf{U})\,\mathrm{d}t \tag{40}$$

where

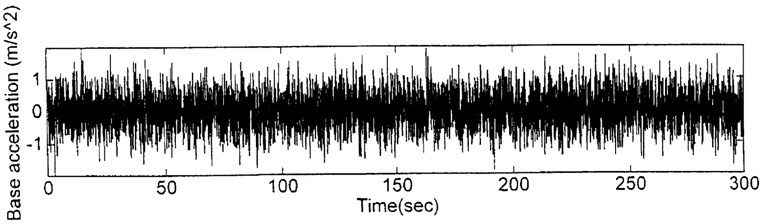


Fig. 5. Band-limited white noise base excitation.

Table 1
The maximum values of the structural responses (White noise base excitation)

Floor	Uncontrolled			Decoupling control				LQR			
	Max $\|x\|$	Max $\|v\|$	Max $\|a\|$	Max $\|x\|$	Max $\|v\|$	Max $\|a\|$	Force	Max $\|x\|$	Max $\|v\|$	Max $\|a\|$	Force
1	1.66	13.2	186.4	0.13	3.55	153.6	460.8	0.20	4.72	192.5	411.4
2	3.22	24.1	283.2	0.13	3.55	153.6	719.5	0.28	6.80	271.2	640.2
3	4.58	33.5	345.8	0.13	3.55	153.6	719.5	0.31	7.51	294.5	737.4
4	5.67	42.2	422.9	0.13	3.55	153.6	719.5	0.32	7.72	298.7	772.4
5	6.44	49.4	479.3	0.13	3.55	153.6	841.7	0.33	7.77	298.6	782.3
6	6.83	53.1	509.4	0.31	6.87	211.2	987.2	0.33	7.78	298.3	784.6

Note: In Tables 1 and 2, the units of displacement, velocity and acceleration are cm, cm/s and cm/s^2, respectively, the unit of control force is KN.

$$\mathbf{Q} = \mathrm{diag}(1.3 \times 10^7)_{12\times 12} \qquad \mathbf{R} = \mathrm{diag}(1.3 \times 10^{-7})_{6\times 6} \tag{41}$$

The numerical simulation is performed on a Pentium 586 PC and the program is written by MATLAB 4.2. Wilson θ method is used to obtain the solutions of the governing equations. The CPU time corresponding to the proposed control law and the LQR control law are 76 and 84 s, respectively, which suggests that the proposed control law holds higher computational efficiency than the LQR. In fact, this result is reasonable because the decoupled MDOF system is modeled as a set of equivalent SDOF systems. For the control design of a SDOF system, the feedback gain of principal regulator has analytic solution, such that the computational time can be reduced significantly. The maximum values of the structural responses corresponding to the uncontrolled and controlled configuration are given in Table 1. It can be seen that the structural responses have been reduced significantly when the control is implemented by the decoupling control law or by the LQR. However, the maximum displacement responses of each floor are different when the control design is based on the LQR. This means that there exists inter-story drift. By the decoupling control design, except for the sixth floor, the maximum response values of each floor are identical. This is because: (a) the structural parameters of each floor of the original structure are the same; (b) the principal regulator parameters of the first five floors are the same. Therefore, the motions of the first five floors are described by the same governing equations. Therefore, there is no inter-story drift among the first five floors so that the safety and integrity of the building have been enhanced by the proposed control law. The inter-story drift between the sixth and the fifth floor can be eliminated by re-carrying out the principal regulator design of the six floor as

$$\gamma_6 = 2.0 \qquad \beta_6 = 2.0374 \tag{42}$$

In this case, the responses of the sixth floor are the same as those of the other floors. Because the design of principal regulator does not affect the responses of the rest floors, this means that the responses of first five floors are unchanged. Therefore, there is no inter-story drift among the floors, while the drift between the ground and the first floor still exists. The primary advantage of the decoupling control law is that the control parameters can be designed independently for the individual floors. It should be noted that the maximum control force is 987.2 KN when the control design is performed by the decoupling control law, which is greater than that of the LQR design. However, this value is acceptable for civil engineering structural control. Fig. 6 shows the comparison of control forces (during the first 12 s) acting on the sixth floor corresponding to the proposed control law and LQR, respectively.

Fig. 7 shows the responses of the top floor which correspond to the uncontrolled and controlled structure (during the first 12 s), respectively, in which the control design is based on the proposed closed-loop decoupling control law. The comparison of control effects respect to the decoupling control and LQR are shown in Fig. 8.

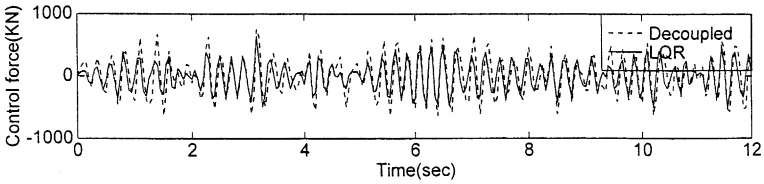
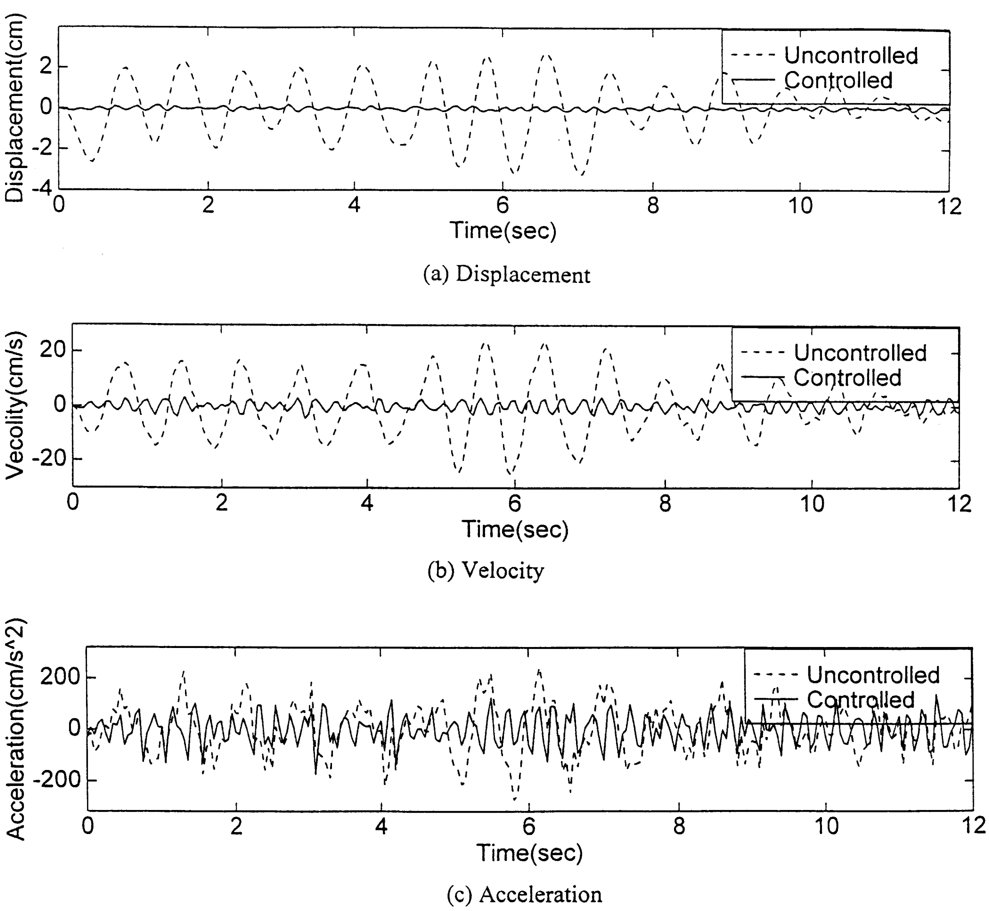


Fig. 6. Comparison of control forces (first 12 s).



(a) Displacement

(b) Velocity

(c) Acceleration

Fig. 7. Responses of the top floor (first 12 s); (a) displacement, (b) velocity, (c) acceleration.

6.2. Example 2: Tianjin earthquake record

Fig. 9 shows the Tianjin earthquake record. The sampling interval is 0.02 s and duration is 11.5 s. Using this record as base excitation, the decoupling control parameters and the weight matrices of LQR are identical as those in Example 1. The comparison of control forces acting on the top floor is shown in Fig. 10.

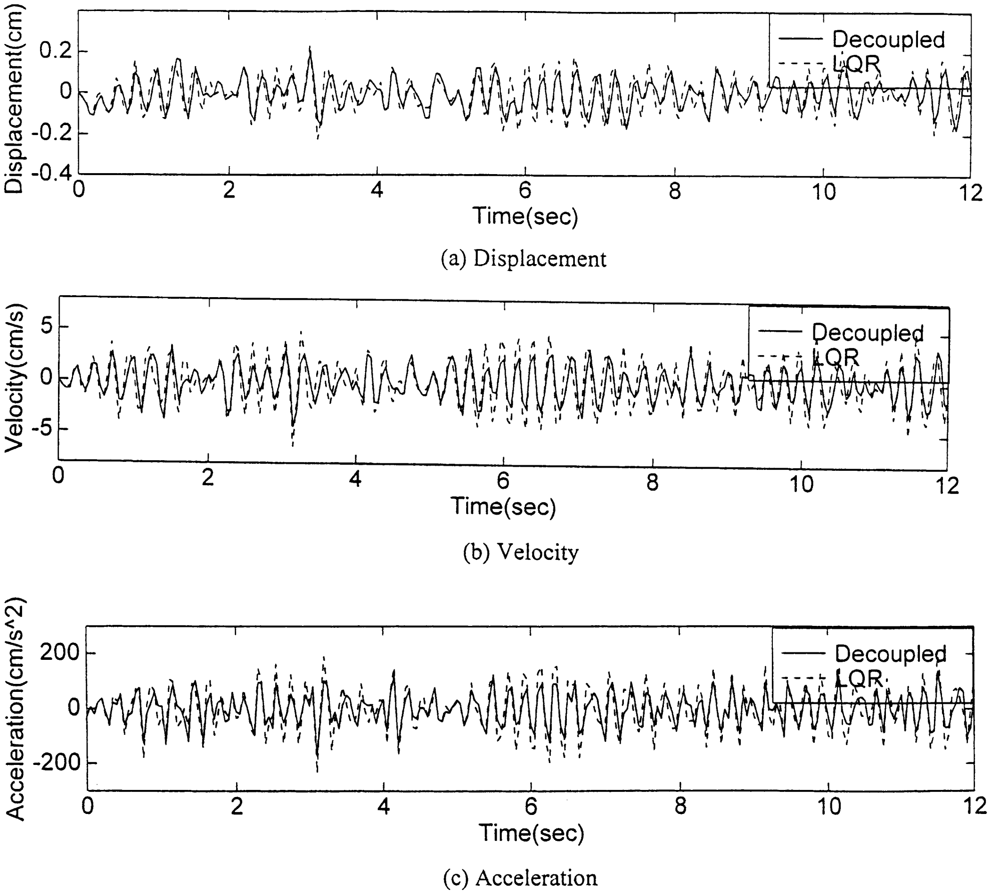


Fig. 8. Comparison of control effects (first 12 s); (a) displacement, (b) velocity, (c) acceleration.

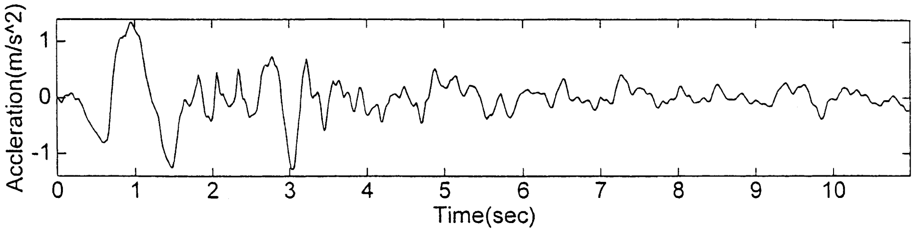


Fig. 9. Tianjin earthquake record (People's Republic of China, 1976).

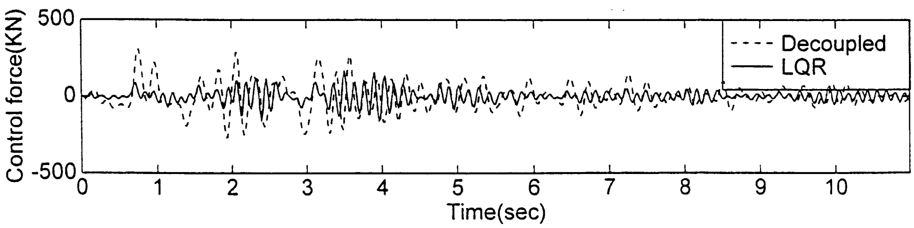
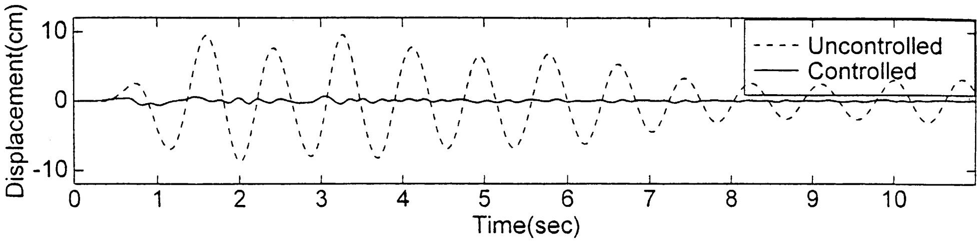


Fig. 10. Comparison of control forces.

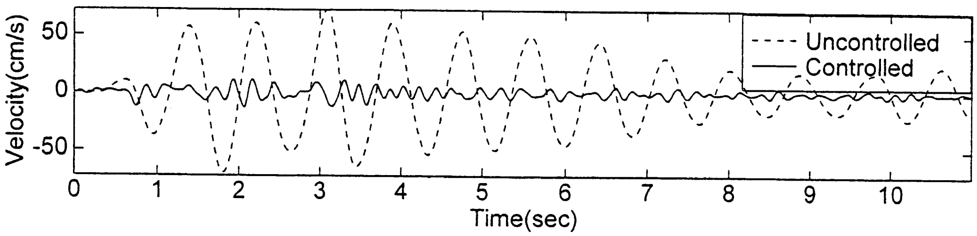
Table 2
The maximum values of the structural responses (Tianjin earthquake wave)

Floor	Uncontrolled			Decoupling control				LQR			
	Max $\|x\|$	Max $\|v\|$	Max $\|a\|$	Max $\|x\|$	Max $\|v\|$	Max $\|a\|$	Force	Max $\|x\|$	Max $\|v\|$	Max $\|a\|$	Force
1	2.37	17.61	140.5	0.07	1.4	56.18	183.5	0.0437	1.23	57.7	103.6
2	4.56	34.42	262.5	0.07	1.4	56.18	244.4	0.0488	1.59	71.4	150.2
3	6.44	49.03	387.7	0.07	1.4	56.18	244.4	0.0483	1.66	74.2	164.4
4	7.94	60.48	500.4	0.07	1.4	56.18	244.4	0.0481	1.67	75.9	167.6
5	8.99	68.25	590.8	0.07	1.4	56.18	280.3	0.0482	1.65	76.3	168.1
6	9.56	72.22	641.1	0.09	1.9	80.19	290.2	0.0484	1.67	76.4	168.2

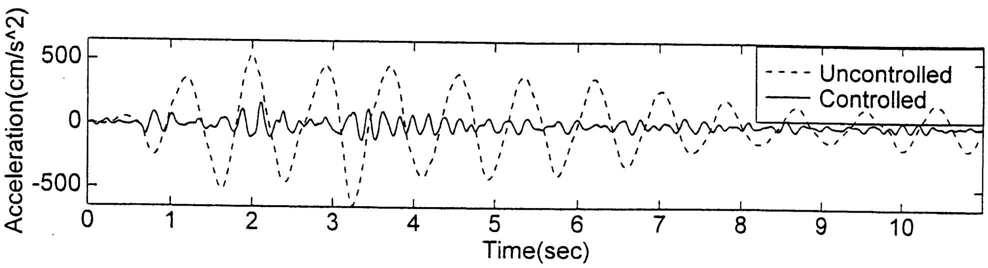
The maximum floor responses and control forces are given in Table 2. The responses of the top floor which correspond to the uncontrolled and controlled structure based on the decoupling control design, and the comparison of control effects respect to the decoupling control law and LQR are shown in Figs. 11



(a) Displacement

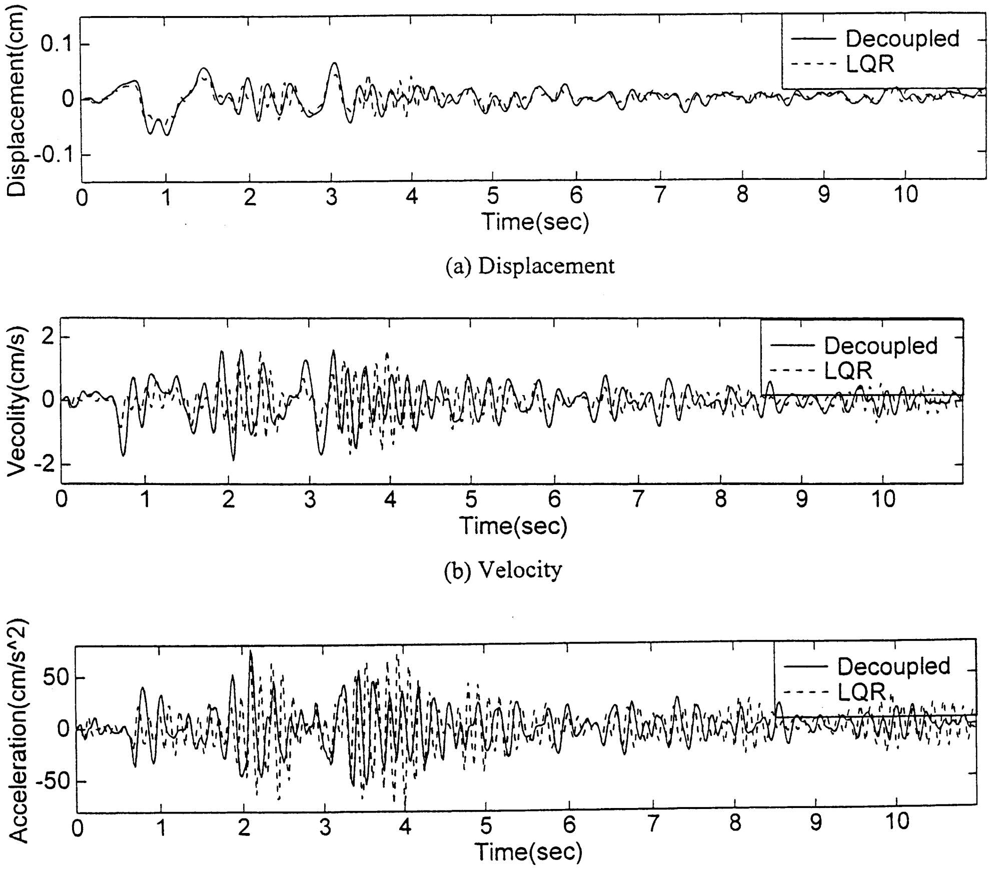


(b) Velocity



(c) Acceleration

Fig. 11. Responses of the top floor; (a) displacement, (b) velocity, (c) acceleration.



(a) Displacement

(b) Velocity

(c) Acceleration

Fig. 12. Comparison of control effects; (a) displacement, (b) velocity, (c) acceleration.

and 12, respectively. Look at Table 2 and Figs. 10–12, the similar conclusions drawn in example 1 can be obtained.

7. Conclusions

In general, it is difficult to realize one to one control strategy for a multi-story structure since the structure is a coupled system. In this paper, the coupled property of a MDOF system has been examined and discussed. It has been pointed out that coupling of a MDOF system forms the positive feedback of the output, possibly increasing the vibration of the structure. Therefore, the vibration of the structure can be suppressed if the couple is "broken off".

In order to realize one to one control of structural vibration, two types of decoupling control laws (open-loop and closed-loop) have been proposed. The advantages of the proposed control laws are that: one to one control can be realized; the control design of each floor can be carried out independently by means of the closed-loop control law. The numerical examples presented in this paper have show that the proposed control law allows the control design of a complex structure to follow the same procedure as employed in design of SDOF systems. Since the control design can be carried out independently for each floor by the use of the closed-loop control law, inter-story drifts have been reduced to zero as illustrated in the numerical examples.

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